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INDETERMINISM, THREE-VALUED LOGIC AND JAN ŁUKASIEWICZ*

In spite of his very critical judgement of philosophy, and modern philosophy in particular, J. Łukasiewicz believes that there is a set of genuine philosophical problems still awaiting their solution. Most of them have already been approached by philosophers of the past. Being critical of the work of predecessors does not mean, in Łukasiewicz's eyes, that one has to reject their achievements. Even today some of these meet all the requirements of serious critical research. Many others, although incorrect, contain a lot of brilliant and exceptional ideas. Hence, every one who intends to resolve a particular philosophical problem has to analyse it first in a large historical context. Philosophy and history of philosophy should never be separated, if one wants to undertake a serious scientific work.[1]

We can see, with regard to the issue of determinism, how thoroughly Łukasiewicz applies those requirements to himself. He begins the analysis of the problem with the presentation of Aristotle's argument contained in chapter 9 of *De Interpretatione*. Yet, we should immediately add that the use he makes of Aristotle's discussion is a particular one. It allows him to put more clearly, and in opposition to Aristotle's, his own point of view. This enables us to proceed in the following way. First, we shall give a short account of the content of chapter 9 of *De Interpretatione*. We shall pay no attention, however, to the contemporary discussion about the question: what did Aristotle really try to say? Secondly, on the basis of that, we shall present Łukasiewicz's interpretation of Aristotle's chapter 9 as well as his own contribution to the solution of the problem. Finally, taking into consideration some more recent works, we shall try to indicate some limits of Łukasiewicz's contribution.

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* This is an abbreviated version of an argument presented in 1990 in Rome under the title: *The Hypothesis of Indeterminism. A Critical Examination of Łukasiewicz's Argument.*

[1] J. Łukasiewicz, *Z historii logiki zdań*, in: *Z zagadnień logiki i filozofii*, Warszawa: PWN 1961, p. 179. There is an English selection of Łukasiewicz's works edited by L. Borkowski: *Selected Works*, Amsterdam: North-Holland Publishing Company and Warszawa: PWN 1980 (hereafter quoted as SW). Cf. also: T. Kwiatkowski, *Jan Łukasiewicz – a Historian of Logic*, Organon 16-17 (1980-81) p. 171; T. Kotarbiński, *Jan Łukasiewicz's Works on the History of Logic*, SL 8 (1958) pp. 61-62.

I. ARISTOTLE AND THE SEA-BATTLE

1. The course of the argument

One of the earliest and most penetrating discussions on determinism is contained in Aristotle's *De Interpretatione*, 9.[2] Aristotle considers the question of whether every true (false) proposition asserting that a certain event has occurred at a certain time, was true (false) before the event in question took place (or failed to take place) at that time. The problem arises, Aristotle argues, when one wants to consider statements about events that are individual in the sense of being tied to a particular moment of time.

> In regard to things present or past, propositions, whether positive or negative, are true of necessity or false (...). When, however, we come to propositions whose subjects are singular terms, while their predicates refer to the future and not to the present or past, then we find that the case is quite changed.[3]

The change becomes evident when we keep in mind one of the most fundamental logical principles, namely, the principle of the excluded middle.[4] Let us assume, for the sake of the argument, that 'p or not-p' is universally true. Then, it will be the case that if someone declares that a certain individual event will take place, and someone else denies it saying that it will not take place, one of them will clearly be making a true statement, while the other a false one. Aristotle's famous example is the case of the sea battle which may or may not occur tomorrow.[5] If we try, in the imagination, to place ourselves in the situation of the Greeks on the eve of that battle, we can say on the strength of *tertium non datur* that it should neither take place nor yet fail to take place tomorrow. Therefore, if we suppose that the first possibility happens to obtain, then it is already true that there will not be a sea battle tomorrow. By the same token, if we suppose that the sea battle will take place tomorrow, then it will be false today to say that there will not be a sea battle tomorrow. Thus, the unrestricted applicability of the principle of the excluded middle to the statements about future contingent events seems to commit us to holding that all future events are predetermined. The result of the argument however, in Aristotle's opinion, is at least a very strange one: it seems to lead us to determinism and to negation of contingency in the world.

A r i s t o t l e, *On Interpretation*, in: *Works*, London: Harold P Cook, M.A. 1973.

[3] Ibidem p. 131.

[4] We find in Aristotle's writing several slightly different formulations of that principle. For a critical evaluation see, for instance, J.M. B o c h e ń s k i, *Ancient Formal Logic*, Amsterdam: North-Holland Publishing Company 1951, p. 40. In *On Interpretation* we read: "all affirmations and denials must either be true or false" (p. 121).

[5] G. Patzig adds that Aristotle probably thought of the situation of the "evening of 27th September 480 B.C., the evening before the battle of Salamis" See: *Aristotle, Łukasiewicz and the Origins of Many-Valued Logic*, in: *Logic, Methodology and Philosophy of Science IV Proceedings of the Fourth International Congress for Logic, Methodology and Philosophy of Science. Bucharest 1971*, Warszawa: PWN and Amsterdam: North-Holland 1977, p. 921.

> This and other strange consequences follow, provided we assume in the case of a pair of contradictory opposites (...) that one must be true, the other false, that contingency there can be none and that all things that are or take place come about in the world of necessity.[6]

The deterministic conclusions however, are not the ones Aristotle could accept. He tries to avoid them by evoking the fact of everyday experience. This is what he says:

> We know from our personal experience that future events may depend on the counsels of action of men, and that, speaking more broadly, those things that are not uninterruptedly actual exhibit a potentiality, that is, a 'may or may not be' (...).[7]

We may take for granted that for Aristotle the occurrence of a sea battle tomorrow is a contingent fact. For in similar circumstances in the past and in the future it sometimes is true and sometimes false to say 'a sea battle will take place tomorrow'

Proceeding dialectically, that is, explaining first the arguments for what we may call the 'deterministic view' and then for the 'indeterministic', Aristotle comes to an *aporia* to be solved.[8] The solution he gives is not likely to be interpreted uneqivocally. Since the exact wording of Aristotle's formulation is important, we quote again:

> That is, all things must be or not be, or must come or not come into being, at this or that time in the future. But we cannot determinately say which alternative must come to pass. For example, a sea-fight must either take place on the morrow or not. No necessity is there, however, that it should come to pass or should not. What is necessary is that it either should happen tomorrow or not. And so, as the truth of propositions consists in corresponding with facts, it is clear in the case of events where contingency or potentiality in opposite directions is found that the two contradictory statements about them will have the same character. With what is not always existent or not at all times non-existent we find this exactly the case.[9]

The quoted text as well as the whole of chapter 9 in *De Interpretatione* is far from being clear. No wonder, therefore, that it gave rise to a real controversy focused on the issue: what did Aristotle want to say? Many answers have been given. Łukasiewicz has taken Aristotle to hold the position that one may accept the principle of the excluded middle for all statements (even those concerned with future contingent events), but not accept the validity of the principle of bivalence for the statements concerned with future events.[10] More rigorously, what Aristotle

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[6] Aristotle, *On Interpretation*, p. 137.

[7] Ibidem p. 137, 139.

[8] See: J. Hintinkka, *The Once and Future Sea Fight. Aristotle's Discussion of Future Contingents*, in: *Time and Necessity*, Oxford: Clarendon Press 1973, p. 154.

[9] Aristotle, *On Interpretation*, p. 139, 141.

[10] That Aristotle does not distinguish by name these two principles was probably clear to Łukasiewicz. He held, however, that this distinction, in some sense, is present in Aristotle's text. D. Frede (*Aristoteles und die Seeschlacht. Das Problem der Contingentia Futura in 'De Interpretatione 9'* (Hypomnemata 27), Göttingen: Vandenoeck and Ruprecht 1970) says that it is impossible to derive such an interpretation from Aristotle's Greek text. The reason is that Aristotle has nowhere ascribed truth to molecular propositions such as 'p or q' or 'p or non-q'

says in *De Interpretatione 9* (always according to Łukasiewicz) can be put along the following lines:

1. Aristotle was familiar with the semantic formulation of the principle of the excluded middle. For the sake of clarity Łukasiewicz calls it the law of bivalence. It states that "every proposition is either true or false" [11]

2. Aristotle believed that the law of bivalence, provided that the classical definition of truth holds, inevitably leads to determinism.

3. Though determinism, in Aristotle's opinion, was a legitimate consequence of the law of bivalence, he found himself unable to accept the deterministic doctrine.

4. Hence he was forced to restrict the validity of the law of bivalence with regard to statements about the future.

2. Łukasiewicz's interpretation of Aristotle

Referring to the passage of Aristotle we have quoted above Łukasiewicz tries to express the Stagirite's thoughts in this way:

> In the famous chapter 9 of 'De Interpretatione' Aristotle seems to have reached the conclusion that the alternative 'either there will be a sea battle tomorrow or there will not be a sea battle tomorrow' is already true and necessary today, but it is neither true today that 'there will be a sea battle tomorrow' nor that 'there will not be a sea battle tomorrow' These sentences concern future contingent events and as such they are neither true nor false today.[12]

The alternative 'either there will be a sea battle tomorrow or there will not be a sea battle tomorrow' is logically a true proposition (an alternative is false only when both of its arguments are false); hence it is logically necessary. However, it is not necessary that 'there will be a sea battle tomorrow' and, similarly, it is not necessary that 'there will not be a sea battle tomorrow' Therefore, the conclusion that 'there will be a sea battle tomorrow' or 'there will not be a sea battle tomorrow' is not quite correct. From the assumption that 'it is true that p or not-p' does not follow 'either it is true that p, or it is true that not-p' Such a conclusion is valid only if we accept the principle that every proposition is either true or false. In Łukasiewicz's opinion, however, Aristotle did not accept the unrestricted validity of that principle. It was because of his anti-deterministic position. He argued that the doctrine of determinism is fully consonant with the principle of the excluded middle. Moreover, he had no doubts that the inference from *tertium non datur* to determinism is valid. On the other hand, however, the Stagirite was fully aware of the fact that the so-called 'propositions of two-sided possibility' belong to the description of the world. Hence, according to common sense, determinism must be false. In this case, Łukasiewicz argues, there was no other way for Aristotle as the one calling in question the principle according to which all propositions are either true or false. The principle of bivalence, one of the basic principles of our entire

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[11] *Uwagi filozoficzne o wielowartościowych systemach rachunku zdań*, in: *Z zagadnień*, p. 161 (SW, p. 176).

[12] *O determinizmie*, in: *Z zagadnień*, p. 125 (SW, p. 125).

logic, is not as obvious as it seems to be. The merit of having noticed that belongs to Aristotle. Trying to support this way of reading Aristotle's text Łukasiewicz points to the fact that "this was the interpretation of Aristotle given by the Stoics, who, being determinists, disputed his view, and by the Epicureans, who defended indeterminism and Aristotle" [13]

In fact, the Stoic logic of propositions was a *two-valued* logic.[14] Chrysippus, an "outspoken determinist", established the law of bivalence as the fundamental principle of Stoic dialectic.[15] This principle, Łukasiewicz argues, was held in conscious opposition to the view wide-spread among the Epicureans who firmly believed in the indeterministic *Weltanschauung*. Cicero testifies that the Epicureans hold that propositions about future, contingent events are neither true nor false.[16] According to some information transmitted by Boethius, the Stoics ascribed this Epicurean view to Aristotle. This, in turn, brought about that the Peripatetics, when trying to defend Aristotle against this objection, introduced a somewhat curious distinction between the proposition *definite vera vel falsa* and the proposition *indefinite vera vel falsa*. Boethius assures us that though it cannot be found in Aristotle in this exact wording, it results from the context.[17] It is certain that the above mentioned distinction was meant to solve the problems connected with the law of bivalence, that is, the principle stating that 'every proposition is either true or false' Boethius succeeded in imposing the conception of the proposition *vera vel falsa* but *indeterminate* upon the scholastic tradition.[18] The distinction brought no contribution to the solution of the problem of determinism.[19]

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[13] Ibidem.

[14] J.T Clark, *Conventional Logic and Modern Logic. A Prelude to Transition*, Woodstock: College Press 1952, pp. 23-24; Łukasiewicz, *Z historii logiki zdań*, in: *Z zagadnień*, p. 182 (SW, p. 207).

[15] "Concludit enim Chrisippus hoc modo: si est motus sine causa, noc omnis enuntiatio quod *aksióma* dialectici appellant, aut vera aut falsa est; causas enim efficientis quod non habebit, id nec verum nec falsum erit. Omnis enim enuntiatio aut vera aut falsa est. Motus ergo sine causa nullus est (...)", Cicero, *De fato*, 20, in: *Scripta que manserunt omnia*, Stuttgart: B.G. Teubner 1965.

[16] Ibidem p. 37

[17] "Concludit igitur totam de futuris et contingentibus propositionibus questionem et ait: manifestum esse non necesse esse omnes affirmationes et negationes definita veras esse (sed dust definite atque ideo subaudiendum est)" M.S. Boethius, *Comentarii in librum Aristotelis Peri Hermeneias*, P I, Lipsiae 1877 (editio prima), col. 340, p. 125.

[18] Klósak points out that this distinction can be find in: St. Albertus (*Liber I Peri Hermeneias*, tract. V, c. 4), St. Thomas Aquinas (*In libros Peri Hermeneias expositio*, lib. 1, cap. 9, lectiones 13-15), and today in J. Maritain (*Elémentes de philosophie*, vol. 2). See: K. Klósak, *Teoria indeterminizmu ontologicznego a trójwartościowa logika zdań prof. Jana Łukasiewicza*, AK 49 (1948) pp. 215-216.

[19] Boethius' distinction between affirmatio vel negatio *definite vera vel falsa* and affirmatio vel negatio *indefinite vera vel falsa* seems to be a rather muddled one. For what does it mean that the proposition about a future event is now true or false, but *indefinite*? In fact, there is only one possibility Such propositions are either determined with regard to the truth, and therefore *verae vel falsae determinate*, or they are not yet determined, and then, precisely speaking they are not *verae vel falsae indeterminate* but *indeterminatae quoad veritatem vel falsitatem*. Thus, the distinction given by Boethius turns out to be rather a verbal one.

Yet now it testifies, at least in Łukasiewicz's eyes, that Aristotle tried to discredit the deterministic doctrine by pointing to the limits of the principle of bivalence.

II. THE HYPOTHESIS OF INDETERMINISM FROM A LOGICAL POINT OF VIEW

1. Between logic and ontology

It is a commonplace to assert that Łukasiewicz assimilated Aristotle's argument against determinism, making it his own in order to support the three-valued logic.[20] The author responsible for spreading that opinion was probably M. Schlick. In the paper published in 1931 in *Die Naturwissenschaften*[21] he used the term "logical determinism" for the position Aristotle was likely to have in chapter 9 of *De Interpretatione*. "Logical determinism", wrote Schlick, claims that "the principles of contradiction and excluded middle would not rank as statements about future states-of-affairs if determinism did not prevail" Then, he added: "Even nowadays this argument is at times held to be coercive, and has actually been made the basis for a new kind of logic" [22] The title of Łukasiewicz's article, *Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls*, annotated by Schlick among references leaves little doubt that, in his opinion, Łukasiewicz was a modern defender of 'logical indeterminism' Since the time of Schlick this opinion has been often repeated. F Waismann, for instance, referring to the doctrine of 'logical determinism' as well as to Schlick's paper writes: "This kind of argument was actually propounded by Łukasiewicz in favour of a three-valued logic with 'possible' as a third truth-value alongside 'true' and 'false' " [23]

According to yet another opinion, slightly different, Łukasiewicz regarded the discovery of many-valued logic as a solution of the old controversy determinism-indeterminism.[24]

In our view both of them are apparently mistaken. For both share a certain presupposition which is at the root of our problem and which, we think, was wrongly ascribed to Łukasiewicz by Schlick and his successors. The presuppo-

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[20] Recently S. H a a c k repeats a similar idea in the book *Philosophy of Logics*, Cambridge: University Press 1988, pp. 208-209.

[21] *Die Kausalität in der gegenwärtigen Physik*, Die Naturwissenschaften 19 (1931) pp. 145-162. English translation under the title *Causality in Contemporary Physics* appeared in M. S c h l i c k, *Philosophical Papers*, vol. II (1. ed., vol. I-II, 1925-1936), ed. by H. M u l d e r and B.F.B. van de V e l d e - S c h l i c k, Amsterdam: D. Reider Publishing Company 1979, pp. 176-209.

[22] Ibidem p. 202. See also: *Causality in Everyday Life and in Recent Science*, in: *Philosophical Papers*, p. 250.

[23] *How I See Philosophy*, in: *Contemporary British Philosophy Personal Statements*, ed. H.D. L e w i s, London: Allen and Unwin Ltd. 1956, p. 456.

[24] K ł ó s a k, *Teoria indeterminizmu*, p. 209, 218; also: J. G i e r a s i m i u k, *Racjonalistyczno-fenomenalistyczna i materialistyczna koncepcja związku przyczynowego a indeterminizm*, SF 5 (1977) p. 108.

sition is that the issue of determinism, a problem about the real world, might be solved by purely logical analysis. Łukasiewicz would never agree with that supposition. He suspected that there is a relation between the principle of bivalence and the controversy determinism-indeterminism. We may also say that in his mind the issue of determinism was somewhat associated with the possibility of constructing different systems of logic. But he would never accept the view of logical determinism. In the paper quoted by Schlick[25] he wrote that the controversy around the principle of bivalence has a 'metaphysical background'; the supporters of the law of bivalence are usually 'decided determinists', while its opponents tend towards indeterminism. Personal interests of the inquirers which inspire their investigations cannot, however, decide the sort of real controversy. The suspected connections (if there are any), which link the law of bivalence with strict determinism on the one hand and indeterminism with the restricted validity of that law on the other hand, certainly deserve a careful exploration. Łukasiewicz believed that he would be able to elucidate the problem in its logical implications. But in the article mentioned above he does not give any satisfactory direct answer. Therefore, in order to specify the nature of the link between the law of bivalence and determinism or indeterminism, we must take into consideration a much broader context of Łukasiewicz's work.

In one of his first papers[26] Łukasiewicz raises the question whether from the logical relation between an antecedent and a consequent can be inferred the occurrence of the states of affairs asserted by that relation. His answer is straightforwardly negative. The nature of the relation between different states of affairs is causal or factual; it cannot be inferred from the logical relation pertaining to propositions or statements. Otherwise, the best method to verify the occurrence of a causal link would be the examination of the logical relation between propositions asserting that link. This, however, cannot be true for the causal link occurs among real objects, whereas the logical relation occurs among abstract objects. The former cannot be reduced to the latter and the opposite holds as well.[27] Causal relationships and logical relations apply to objects of different sort.

On the other hand, when we analyse these two relations more carefully, they also reveal some common features. We can even establish a certain logical connection between causal relationship and inference relation: it might be called asymmetry. While the logical relation between statements does not allow us to say anything about the causal connection pertaining to states of affairs asserted by those statements, the opposite is likely to be true. The causal link between states of affairs (events or things) allows us to establish the logical relation between propositions or sentences referring to those states of affairs. That is simply because these two kinds of connections, the first linking the cause with the effect and the second

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[25] *Philosophische Bemerkungen*, Comptes rendus de la Société des Sciences et des Lettres de Varsovie 23 (1930) pp. 51-77; reprinted in: *Z zagadnień*, pp. 144-163 (SW, pp. 153-178).

[26] *Analiza i konstrukcja pojęcia przyczyny*, in: *Z zagadnień*, pp. 9-62.

[27] Ibidem pp. 33-37

linking the antecedent with the consequent, have at least one thing in common: both are *necessary*. The meaning of the word 'necessary' is different in both cases; yet it indicates that both the relationship between cause and effect and the relation between premise and conclusion share certain formal characteristics. Owing to that, the causal link can be set forth in the form of the causal implication, while the logical connection can be expressed by means of the formal implication.[28] With the help of Łukasiewicz's symbolism we can enhance the conformity as well as the difference between the causal and the formal implication saying that the thesis:

$$CCpqNKpNq,$$

holds for both of them, while the thesis:

$$CNKpNqCpq,$$

holds only for the logical implication linking the antecedent with the consequent.[29] If p implies q in the sense of a causal implication, then between p and q the formal implication also holds. The opposite, however, does not turn out to be true. For if *Cpq* stands for the formal implication, then from *Cpq* it is not possible to infer the causal implication occurring between p and q. Had this been the case we should have accepted *CNKpNqCpq* which, for the causal relation, is to be rejected.

Between logic and ontology there is a world of difference. Łukasiewicz, as we can see, uses a lot of ink and energy to stress that point. If he tries to set out the problem of determinism in logical terms, he does it because of the conviction that logic may throw some light on that problem. But the alleged 'logical determinism' or 'logical indeterminism' of the author of *Philosophische Bemerkungen* is no more than a misunderstanding of Łukasiewicz's intentions and arguments.

2. The hypothesis of indeterminism

In *Philosophische Bemerkungen*, the founder of many-valued logic, pointing to the connection between determinism and the law of bivalence, referred to Aristotle and to the Stoics. Yet he was not likely to accept the standpoint of Aristotle who held that determinism results from the law of bivalence, provided the classical definition of the truth is correct. He also rejected the opinion professed by the Stoics according to which strict determinism and the law of bivalence are inferentially equivalent. Łukasiewicz's standpoint essentially differs both from that of the Stagirite and that held by the Stoics. The difference becomes clear from the very beginning. We remember that both Aristotle and the Stoics singled out the principle of the excluded middle (in a semantic formula) as the first premise of their argumentation in favour of, or against, determinism. This is not

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[28] By 'formal' implication I mean what the logicians usually call the 'material' implication.

[29] In ordinary speech the first thesis is read: 'if (if p then q) then it is not the case that p and not-q'; and the second: 'if it is not the case that p and not-q, then if p then q' See: Z. J o r d a n, *O logicznym determinizmie*, SL 14 (1963) p. 70.

the case of Łukasiewicz who begins with an ontological supposition,[30] that is, with the hypothesis of indeterminism. According to this supposition not every event in the world is uneqivocally determined and settled in the string of events or things existing *ab aeterno*. There is at least one event in the universe,[31] preceded by a sequence of other events, whose causes do not reach from 'all eternity' An affirmation about that event before its occurring is not true, while a negation about the same event before its occurring is not false. Therefore, the principle that every proposition is either true or false cannot be held with respect to all the events in the world.[32]

We can see that according to Łukasiewicz the hypothesis of indeterminism is a supposition about the existence of propositions which are neither true nor false. To suppose that indeterminism is true means to suppose that besides the true and the false propositions, or sentences, there are 'third' sentences, 'neutral' or 'indeterminate' propositions.[33] The true propositions are necessary (in the logical sense of the word) so that the false propositions are logically impossible. What about the 'third' sentences, 'neutral' propositions? From the logical point of view they are neither necessary nor impossible: they lie in the area of *two-sided* possibility.

Again, it is clear that the value of the hypothesis of determinism cannot be proved by means of logic. The hypothesis of indeterminism is an ontological hypothesis. Therefore, it is the task of ontology to prove whether the hypothesis of indeterminism is consonant with reality or not. Taken this for granted we may question the significance of Łukasiewicz's logical investigations. In other words our concern now is with the problem whether the discovery of three-valued logic brings about real progress with regard to the issue determinism-indeterminism.

3. The formal basis of indeterminism

As we know Łukasiewicz often reverted to the connection between the many-valued logic and the problem of determinism. On the other hand, he never failed to underline the fundamental difference between logic and ontology. How could it be reconciled? We touch here the point which explains why the founder of many-valued logic dedicated so much attention to Aristotle and especially to

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[30] Even in the text quoted by Schlick, Łukasiewicz writes: "I can *assume without contradiction...*" (emphasis by me); *Z zagadnień*, p. 153 (SW, p.165). Cf. also: *Elementy logiki matematycznej*, Warszawa: PWN 1958, p. 68.

[31] We may speak on 'one event' for Łukasiewicz opposes indeterminism to *strict* determinism laying down that *every event* is to be predictable. Cf. K.R. P o p p e r, *The Open Universe. An Argument for Indeterminism*, London: Hutchinson 1988, p. 6.

[32] The same kind of reasoning we may effect after having assumed the hypothesis of determinism. In this case, both the past and the future are settled once and for ever. Consequently, every proposition would be either true or false. Moreover, every true proposition would be logically necessary and every false proposition logically impossible. For one cannot assume that *V'p* (it is true that p) and say at the same time that *Np* (not-p), and vice versa.

[33] Ł u k a s i e w i c z, *Elementy logiki matematycznej*, p. 68.

chapter 9 of *De Interpretatione*. In Łukasiewicz's opinion Aristotle made the first step on the path leading to the solution of the problem of determinism; he singled out the 'third' propositions, that is, the propositions neither true nor false. Łukasiewicz states it clearly:

> I made my assumption (of indeterminism – explanation by me) referring to the authority of Aristotle himself, for no one other than the Stagirite seemed to believe that propositions concerning future fortuitous events are today neither true nor false (...). In stating this the Stagirite tried to avoid determinism, which to him seemed to be unavoidably connected with the principle of bivalence.[34]

Following Aristotle, Łukasiewicz goes much further at the moment he succeeds in building the three-valued logic. The new system of logic becomes a formal basis of the hypothesis of indeterminism. With the new logic we receive a powerful, intellectual tool which enable us to use correctly 'third' propositions. The hypothesis of indeterminism which is the "metaphysical substratum of the new logic"[35] would be a very weak one without any formal basis asserting its formal correctness. The three-valued logic allows us to know when our use of 'third' propositions is contradictory and when it is correct. If we did not have the formal system ruling the use of 'third' propositions, the hypothesis of indeterminism could be rejected because of a purely logical reason: the lack of formal rules governing the use of 'third' propositions. Of course, the formal correctness does not imply the truth of the hypothesis of indeterminism. Łukasiewicz, as we have already seen, was inclined to think that there is only one method of its corroboration: the method confronting the consequences of the indeterministic *Weltanschauung* with the empirical data.[36] Whether or not it is a possible task – that is another question. Łukasiewicz seemed to have some doubts it could be workable at the present stage of our knowledge.[37]

To sum up: with regard to the law of bivalence on the one hand and the controversy determinism versus indeterminism on the other, Łukasiewicz is likely to sustain the following opinions:

1. The unrestricted validity of the law of bivalence is a formal and necessary (but not sufficient) condition for the hypothesis of strict determinism to be true.[38]

2. The restricted validity of the law of bivalence is a formal and necessary (but not sufficient) condition for the hypothesis of indeterminism to be true.

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[34] *W obronie logistyki*, in: *Z zagadnień*, p. 217-218 (SW, p. 246-247).

[35] *O logice trójwartościowej*, SF 5 (1988) p. 131 (SW, p. 88).

[36] "I have always believed that answer to these questions (whether or not the world is deterministic – explanation by me) can be provided only by empirical data, in the same way that only the empirical data can tell us whether the space in which we move about is Euclidian, or non-Euclidian" See: *W obronie logistyki*, in: *Z zagadnień*, p. 218 (SW, p. 247).

[37] Ibidem. Cf. for instance, the opinion of K.R. Popper who held that 'metaphysical' determinism (that is strict determinism in its ontological version) is simply not falsifiable.

[38] The supporter of logical determinism would mantain that the unrestricted validity of the law of bivalence is a necessary and sufficient condition for the doctrine of strict determinism. As we can see, this is not the case of Łukasiewicz.

Since both are empirical hypotheses this simple, theoretical demarcation does not help in answering the question about their truth or falsity. Nevertheless, the answer to the question whether the limitation imposed upon the law of bivalence involves any absurd consequences is of great value. It brings about a precise formulation of the formal difference existing between determinism and indeterminism; it also permits us to decide from the formal point of view which of the two outlooks is better elaborated. From this perspective the discovery of many-valued logic furnishes a strong formal argument in favour of indeterminism. With the help of Łukasiewicz's logic we cannot say that indeterminism is true, but we can say that from the formal point of view the doctrine of determinism is not better supported than the doctrine of indeterminism.

III. THE CONTROVERSY ABOUT DETERMINISM AND MANY-VALUED LOGIC.

We agree with Łukasiewicz that the doctrine of strict determinism without the valuable support of some kind of "empirical" argument is not better established than the doctrine of indeterminism. Both, then, determinism and indeterminism might be subjected to a further examination as two rival world views. Beginning with that presupposition, in the course of careful analysis, Łukasiewicz singles out certain assumptions on which determinism and indeterminism seem to be based. For the sake of clarity, let us present it with the help of the following schema:[39]

DETERMINISM

(a) The law of bivalence: 'every proposition is either true or false'

(b) The semantic formulation of strict determinism: 'if A is b at time t, it is true at any instant earlier than t that A is b at time t'

(c) The hypothesis of strict determinism, based on a particularly strong formulation of the principle of causality.

INDETERMINISM

(a') The "law of polyvalence": the division of propositions into true and false is not exhaustive; there are propositions neither true nor false.

(b') The semantic formulation of indeterminism: 'if A is b at time t, it is true at any instant earlier than t that A is b at time t for some substitutions of 'A' 'b' and 't'

(c') The hypothesis of indeterminism, based on the "normal" formulation of the principle of causality.[40]

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[39] It is essentially the schema proposed by Z. J o r d a n in his paper *O logicznym determinizmie*, SL 14 (1963) pp. 90-91

[40] We speak about the 'law of polyvalence' in order to avoid the possible misunderstanding that it has to be the law of trivalence ('Every proposition is either true or false or neither true nor false, and *quartum non datur*').

Łukasiewicz points out that (a) and (b) are formally bound, that is, assuming that (a) holds, (b) can be derived from (a) by means of deductive inference. Similarly, on the other hand, if one denies the universal validity of the logical law of bivalence and assumes, for instance, that the principle of trivalence (a') holds, the thesis of indeterminism in semantic formula (b') can be deduced from (a'). This, however, does not prove that the hypothesis of strict determinism, or respectively the hypothesis of indeterminism is true. The force of deductive inference cannot go beyond the semantic level indicated on our schema by (b) and (b'). If the definition (b) refers to a world, it can be only a possible world; it does not say anything about the real world. In order to assume a real and concrete meaning both the semantic formula of strict determinism and the semantic formula of indeterminism have to be *empirically interpreted*. The contact with reality becomes a crucial point now. The investigation of a logician cannot go beyond it and Łukasiewicz knows that. However, he believes that the principle of bivalence and strict determinism on the one hand, and the principle of tri- or polyvalence and indeterminism on the other, are related in some way. He asks: is there any possibility to know in which way they are related? There is no way to get a direct answer to this question. Let us suppose, then, that they are in some way connected. What kind of logical implications can we expect from that? In total disagreement with the position favoured by so-called "logical determinism", as we have already shown, Łukasiewicz reverses the course of the argumentation. This is, in our opinion, one of the most creative points of his analysis.

Let us assume, Łukasiewicz argues, that strict determinism is true. This means that we accept as true the hypothesis of strict determinism. From the premise of strict determinism we can possibly deduce the conclusion that the principle of bivalence is universally valid. Similarly, if we assume that the hypothesis of indeterminism is true, then, starting with that premise, we may arrive at the conclusion concerning the restricted validity of the principle of bivalence. This reasoning gains an additional persuasive force when around 1920 Łukasiewicz builds up his first many-valued logical system: the three-valued logic in which the law of bivalence is not an asserted thesis. Though on every occasion he tries to stress the fact that this discovery does not give any answer to the thesis of determinism, certain connections become obvious from the very outset. Łukasiewicz personally seems to think that, assuming the thesis of indeterminism is true, only a many-valued system of logic can provide an adequate logical apparatus for its further ontological investigations.

One does not need to have much imagination to see that here the status of logic is involved as well as many metaphysical and ontological issues. To call into question the universal validity of the principle of bivalence means to pose a serious challenge to classical logic. Given that there are different many-valued systems, we may ask: In which does the connection, if there is any, of new logical systems with old classical logic consist? Is there just one correct, or true, logical system, or could there be several which are equally correct, or true? What could 'correct'

mean in this context? Does the existence of many-valued systems of logic pose an argument in favour of 'logical relativism'? These and similar questions require much more complete discussion than we are able to present here. Let us only notice that since the time of Łukasiewicz almost all distinguished logicians and philosophers have been involved in these issues.[41]

The supporters of two-valued logic as unique true logic[42] seem to think along the following lines. Different formal logical systems aim to give precise expression to informal arguments. Therefore, ultimately they depend on some non-formalized reasoning which, in turn, involves an intuitive residual logic that is bivalent. In short, reality does not allow us to abandon the universal validity of the principle of bivalence.[43] The proponents of polyvalent logic, by contrast, seem to be impressed by the idea that the inadequacy of two-valued logic becomes apparent when one wishes to confront the formal valid arguments with their informal analogies, which very often turn out to be incorrect. Thus, the adherents of non-classical logic argue that reality appears

to be much richer than classical two-valued logic. From here the idea that different logical systems are applicable to different areas of discourse.[44]

Łukasiewicz himself, as we know, tries very carefully to distinguish logical formalism from its interpretation. His comments with respect to the latter almost always have coincidental character and often seem to disagree with each other. Perhaps we may say that the problem of the status of logic was not always clear to him. And certainly we may say that Łukasiewicz was more interested in establishing the place the principle of bivalence occupies in logic, than in answering the question asking about the status of logic. Analyzing his texts today we can see how carefully he tried to distinguish the law of bivalence ('every proposition is either true or false') from the principle of the excluded middle ('two contradictory propositions cannot be false simultaneously').[45] From the very beginning Łukasiewicz strove to underlie the fact that the principle of bivalence, in the sense he understands it, is "the deepest foundation" of all traditional logic.[46] "Logic changes

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[41] See: M.L. Dalla Chiara Scabia, *Logica*, Milano: A. Mondadori 1980, pp. 32-37

[42] Cf. H. Scholz, *In memoriam Jan Łukasiewicz*, Archiv für mathematische Logik und Grundlagenforschung 3 (1957) pp. 1-18 (reprinted in 1977), states clearly: "Nach meiner Meinung sollte man bis auf weiteres für n > 2 überhaupt nur von n-wertigen Kalkülen sprechen, nicht von n-wertigen Logikkalkülen" (p. 9).

[43] Cf. for example: B. Sobociński, *In Memoriam Jan Łukasiewicz*, Philosophical Studies 6 (1956) p. 31 "The 'fundamental' logic will always be bi-valued. This does not mean that it is a 'form of the mind' in a Kantian sense. It means, instead, that the reality is such that bi-valued logic imposes itself upon us" See also: A. Mostowski, *Recension of Rasiowa's Article. 'Logiki wielowartościowe Łukasiewicza* The Journal of Symbolic Logic 15 (1950) p. 223.

[44] A classical example is the so-called 'quantum logic' See the 1948 autumn issue of Dialectica 2 (1948); Cf. also: H. Putnam, *Three-Valued Logic*, Philosophical Studies 8 (1957) pp. 73-80; *Is Logic Empirical?*, Boston Studies in the Philosophy of Science 5 (1969) pp. 216-241

[45] *Uwagi filozoficzne o wielowartościowych systemach rachunku zdań*, in: *Z zagadnień*, pp. 154-155 (SW, pp. 164-165).

[46] *W obronie logistyki*, in: *Z zagadnień*, p. 217 (SW, p. 246). In 1947 Łukasiewicz came to the conclusion that "a thesis which embodied this principle might be taken a a single axiom of the whole

from its very foundations", if we assume that in addition to truth and falsehood, there is also some third logical value, or even several such values. But shall we accept such an assumption? Łukasiewicz was convinced that it would hardly be possible to prove its validity by logical means. On the other hand, it is also difficult to accept the principle of bivalence as self-evident since one may doubt its universal validity.

Today we know that many problems connected with logical principles might disappear when we realize that while the law of bivalence is a metalogical principle, i.e., it belongs to the metatheory of logic, the principle of the excluded middle is the logical law of certain logical systems.[47] When Łukasiewicz stated the principle of bivalence for the first time, he was not fully aware of the distinction between logic and meta-logic.[48] Later, when he understood it, he voiced the opinion that the construction of many-valued systems of logic depends on the decision concerning metalogical items. This opinion finds its corroboration in the posterior development of the reflection upon logic.[49] P. Rutz, in a dissertation *Zweiwertige und mehrwertige Logik*, contributes two theorems, both providing convincing support to the idea of the unity of logic.[50] As far as the relation, classical logic – polyvalent logic, is concerned, Rutz states:

1. All propositions of bivalent logic are also valid in any many-valued logic in the case when propositional constants which appear in them are generalized according to some specifically determined meaningful ways.

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propositional calculus" A formal proof of that (Łukasiewicz points out that the sugestion came from Sobociński during his visit to Dublin in 1947) may be found in the article *On Variable Functors of Propositional Arguments*, in: SW, pp. 311-320.

[47] Cf. A.A. Zinoviev, *Filosofskie problemy mnogizacnoj logiki*, Moskva 1960; P Rutz, *Zweiwertige und mehrwertige Logik. Ein Beitrag zur Geschichte und Einheit der Logik*, München: Ehrenwirth Verlag 1973; Rutz, for instance, writes: "(...) viele Unklarheiten bezüglich der mehrwertigen Systeme verschwinden, wenn man beachtet, dass der Zweiwertigkeitssatz (...) ein Metatheorem für bestimte Systeme ist, während das Tertium-non-datur (*ANpp*) ein Satz gewisser logischer Systeme (...) ist" (p. 39).

[48] See: J. Woleński, [Preface to the Polish reedition of Łukasiewicz's first book], *O zasadzie sprzeczności u Arystotelesa*, Warszawa: PWN 1987, p. XXXVII; Also: Sobociński, *In Memoriam Jan Łukasiewicz*, p. 13.

[49] This does not mean that the disagreement about the principle of bivalence can be regarded as belonging to the past. On the contrary, recent development of philosophy was strongly influenced by it. The names of M. Dummett and W Quine are very representative with that regard. The whole issue acquired even more philosophical significance. Thus, for Dummett the rejection of universal validity of the law of bivalence signifies the necessity of embracing an anti-realist position. In fact, Dummett seems to be inclined to abandon realism. See his *Realism*, in: *Truth and Other Enigmas*, Cambridge, Mass.. Harvard University Press 1978, pp. 145-165. Quine agreeing with Dummett that bivalence is "the hallmark of realism" is not, however, prepard to abandon the realist position. Consequently, he holds that the principle of bivalence is universally valid. See his *What Price Bivalence?*, in: *Theories and Things*, Cambridge, Mass.. Harvard Univesity Press 1981, pp. 31-37

[50] Rutz, *Zweiwertige und mehrwertige Logik*, pp. 41-42. The idea that apart from many different logical systems there is one basic LOGIC has been voiced for a long time by J.M. Bocheński. Cf., for instance, *Między logiką a wiarą*, Montricher: Les Editions Noir Sur Blanc 1988, pp. 57-58.

2. To every expression of the n-valued logic corresponds a k-complex expression of classical logic.

The key-point of Rutz's argumentation is the thesis of transitivity: all theorems of two-valud logic can be translated into the formal language of any many-valued logic and vice versa.

This important discussion concerning the status of the principle of bivalence fails to be decisive for the issue of determinism when we realize the following. Łukasiewicz seems to embrace the view that, given the implausibility of strict determinism, we should replace the bivalent logic by tri- or polyvalent logic in the language of science. This postulate appears as a conclusion of the reasoning:

if determinism – then the law of bivalence,
if indeterminism – then restrictions on bivalence,

and leads to an interesting but somewhat bold prediction concerning the application of many-valued logical systems in the language of science. For Łukasiewicz seems to accept the following way of thinking: The two-valued system of traditional logic provides a logical apparatus adequate for the investigations of the ontological structure of the hypothesis of strict determinism. This is because we can express the view of determinism in terms of language whose syntax is derived from bivalent logic. On the other hand, the view of indeterminism can be expressed in terms of language whose syntax is based on a many-valued system of logic. This suggests that the bivalent logical apparatus is no longer suitable if we wish to investigate the ontological structure of the hypothesis of indeterminism; it has to be replaced by a richer logical apparatus based on the "principle of polyvalence"

This postulate is not so certain as it seems to be, for we know that one may call into question the doctrine of determinism by means of arguments entirely based on two-valued logic[51] Moreover, as we have already noted, Łukasiewicz's discussion of both strict determinism and indeterminism is based on the so-called classical or correspondence conception of truth. However, by careful analysis (see the schema above), we discover that the doctrine of strict determinism accepts an additional assumption about the conception of truth. This second assumption has to do with the timelessness or absolute character of truth.[52] A true proposition, if it is true at all, is true once and for all, independently of the time at which it is expressed. Similarly, a false proposition, if it is false at all, is false once and for ever, independently of the time of its utterance. The absolute character of truth when accepted fully justifies the need of many-valued logic for the investigation of indeterminism. If we, however, reject the absolute character of truth and accept the

[51] This is also the case of Łukasiewicz when we think about his refutation of argument supporting the doctrine of strict determinism. See: *O determinizmie*, in: SW, pp. 110-128.

[52] This has been pointed out by Z. Jordan. See: *O logicznym determinizmie*, SL 14 (1963) p. 92.

principle of temporal relativity of truth,[53] then it becomes doubtful whether a logical apparatus adequate for the investigations of the indeterministic ontological structure must be polyvalent.

INDETERMINIZM, LOGIKA TRÓJWARTOŚCIOWA I JAN ŁUKASIEWICZ

Streszczenie

Celem artykułu jest przedstawienie związków między ontologiczną hipotezą indeterminizmu i logiką trójwartościową w pismach twórcy logik wielowartościowych – Jana Łukasiewicza.

Według rozpowszechnionej opinii autorstwa M. Schlicka i F. Waismanna, Jan Łukasiewicz był zwolennikiem doktryny „logicznego indeterminizmu" głoszącej, że zasada sprzeczności i zasada wyłącznego środka wystarczają do uzasadnienia ontologicznej hipotezy indeterminizmu. Analiza tekstów polskiego logika ukazuje, że opinia ta jest nieuzasadniona. Łukasiewicz zawsze podkreślał odrębność obszarów: formalnego i ontologicznego. Równocześnie jednak był świadom istniejących między nimi związków. Twórca logiki trójwartościowej pytał: czy fakt powstania nieklasycznych logik ma jakieś znaczenie dla sporu: determinizm – indeterminizm? Odpowiadał, że logika trójwartościowa stanowi formalną bazę dla hipotezy indeterminizmu, a logika dwuwartościowa – dla determinizmu. W rozumowaniu Łukasiewicza daje się wykryć następujący kierunek: jeśli przyjmiemy indeterminizm (którego prawdziwość trzeba by jeszcze udowodnić na empirycznej drodze), to logika trójwartościowa naturalnie jawi się jako formalna baza tej hipotezy. Pozytywna ocena logik nieklasycznych nie wynikała z przekonania, że na ich podstawie można udowodnić słuszność tezy indeterminizmu, ale z przekonania, że bez takiej bazy formalnej doktrynę indeterminizmu trudniej byłoby przedstawić w sposób zrozumiały.

Dokładniejsza analiza pokazuje, że nawet to ostrożne stanowisko idzie zbyt daleko. Okazuje się, że związki uznane przez Łukasiewicza domagają się jeszcze innego założenia dotyczącego koncepcji prawdy. Łukasiewicz przyjął milcząco semantyczną koncepcję prawdy absolutnej, która później stała się przedmiotem ożywionej i ciągle dalekiej od zakończenia dyskusji.

[53] As it seems to be the case of Łukasiewicz himself. See: J. Woleński, P Simons, *De Veritate: Austro-Polish Contributions to the Theory of Truth from Brentano to Tarski*, in: K. Szaniawski (ed.), *The Vienna Circle and the Lvov-Warsaw School*, Dordrecht: Kluver Academic Publishers 1989, pp. 400-401.